Continuity Equation for the Flow of Fisher Information in Wave Scattering [1]

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Background

LIGO Livingston

LIGO is a Michelson interferometer that detects gravitational waves by precisely estimating the relative laser phase shifts

- Precision experiments estimate a parameter θ based on measurement data X
- **θ can represent different system parameters,** such as:
  - Phase shifts in gravitational wave detection[2]
  - Position/Angle of a particle in levitated optomechanics[3]
- The key quantity is the **Fisher Information (FI)**

$$I(\theta) = \mathrm{E}_\theta\left[\left(\frac{\partial}{\partial\theta}\log p(X;\theta)\right)^2\right]$$

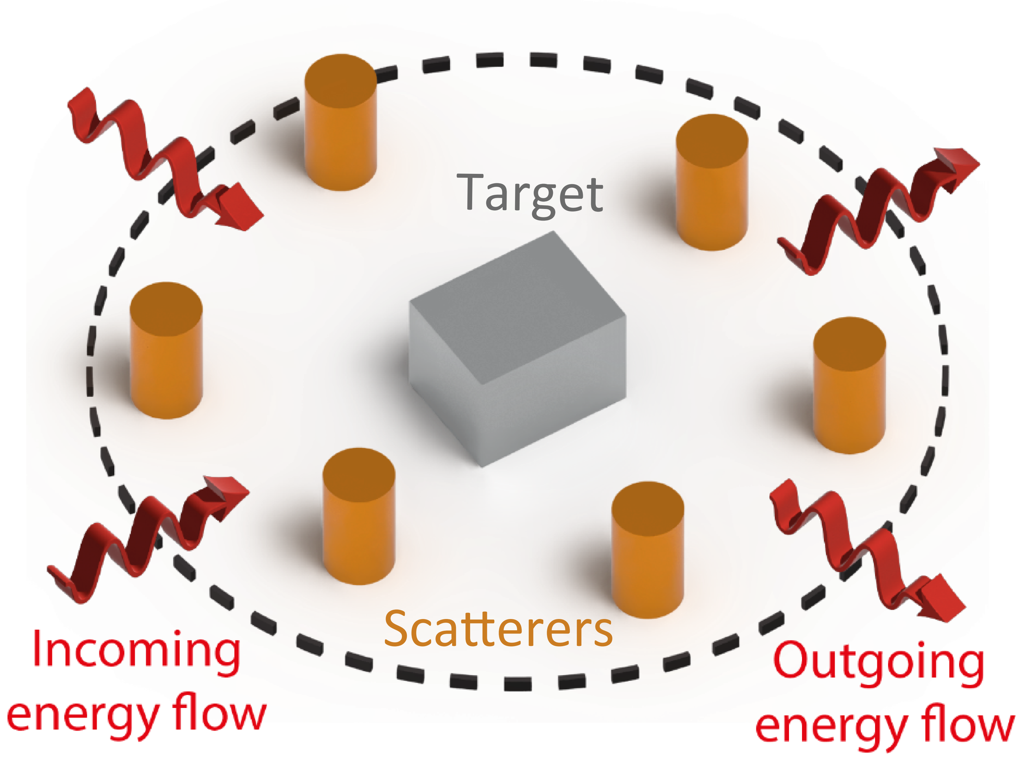
- FI measures the information content of a signal about the parameter θ.
- **Higher FI enables more precise measurements**

Motivation

- Maximizing Fisher Information (FI) is essential for high-precision experiments.
- Inverse design requires understanding how system modifications influence FI at the detector.

How is Fisher Information transferred from the system to the detector?

Energy viewpoint

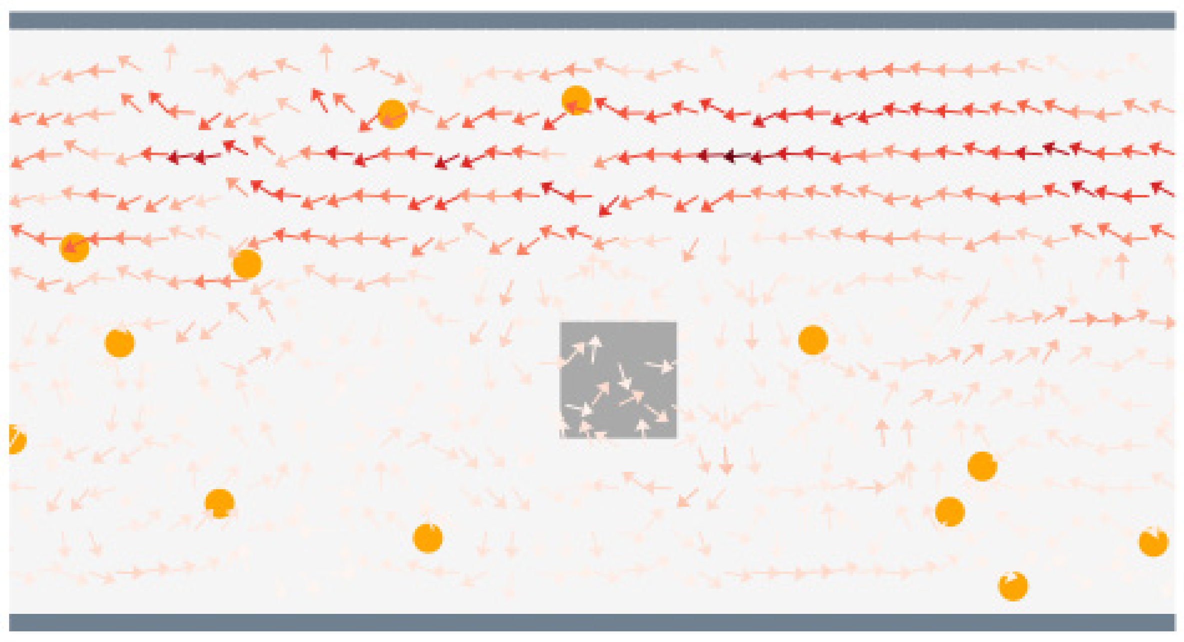


Energy flux

$$\boldsymbol{S}^P = \mathrm{Re}\,(\boldsymbol{E}_\omega^* \times \boldsymbol{H}_\omega)/2$$

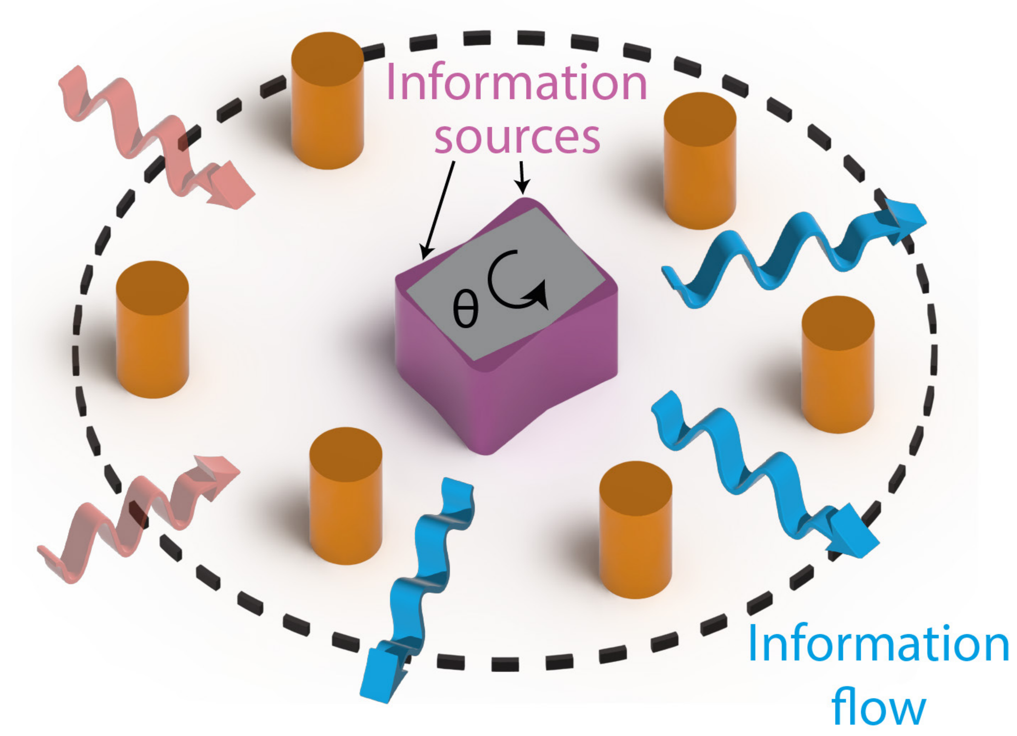
Energy conservation

$$\nabla \cdot \boldsymbol{S}^P + \partial_t u = \sigma$$



The energy of the electromagnetic wave is transmitted through the complex scatterin system (orange circles), as shown by the Poynting vector field (red arrows) and interacts only weakly with the target (grey square).

Fisher Information viewpoint



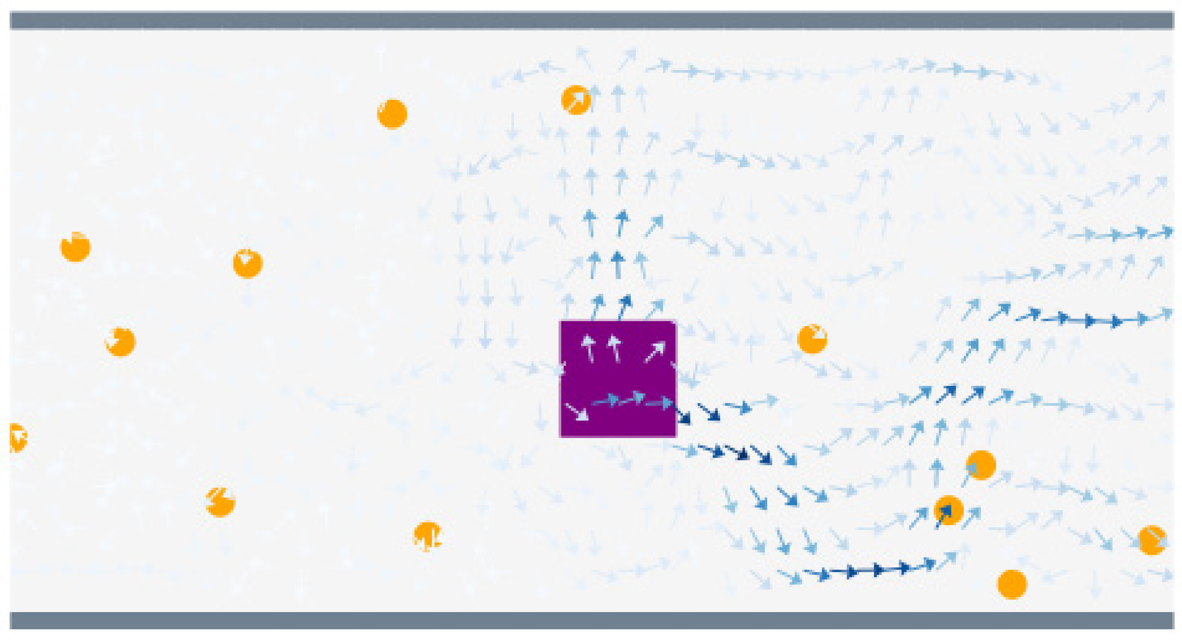
Fisher Information flux

$$\boldsymbol{S}^{FI} = 2\,\mathrm{Re}\,(\partial_\theta \boldsymbol{E}_\omega^* \times \partial_\theta \boldsymbol{H}_\omega)/(\hbar\omega)$$

Information conservation

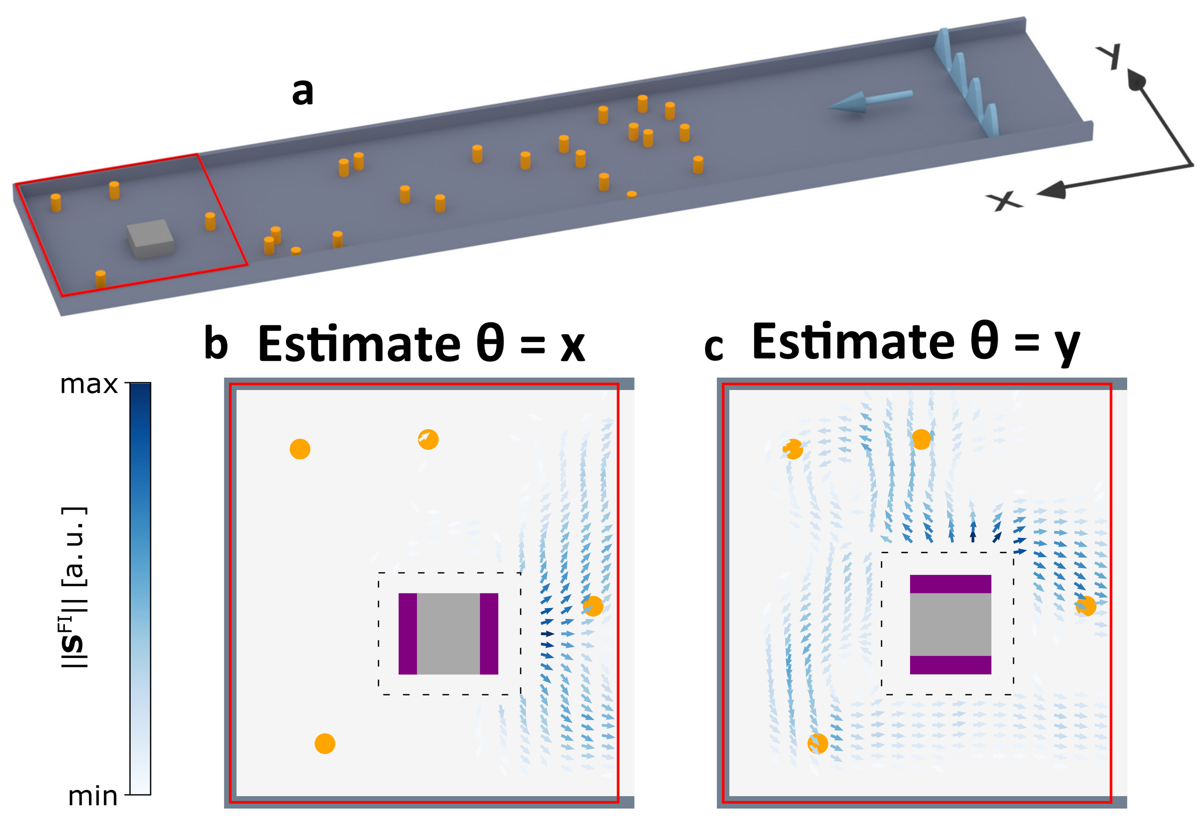
$$\nabla \cdot \boldsymbol{S}^{FI} + \partial_t u^{FI} = \sigma^{FI}$$

FI flows like energy



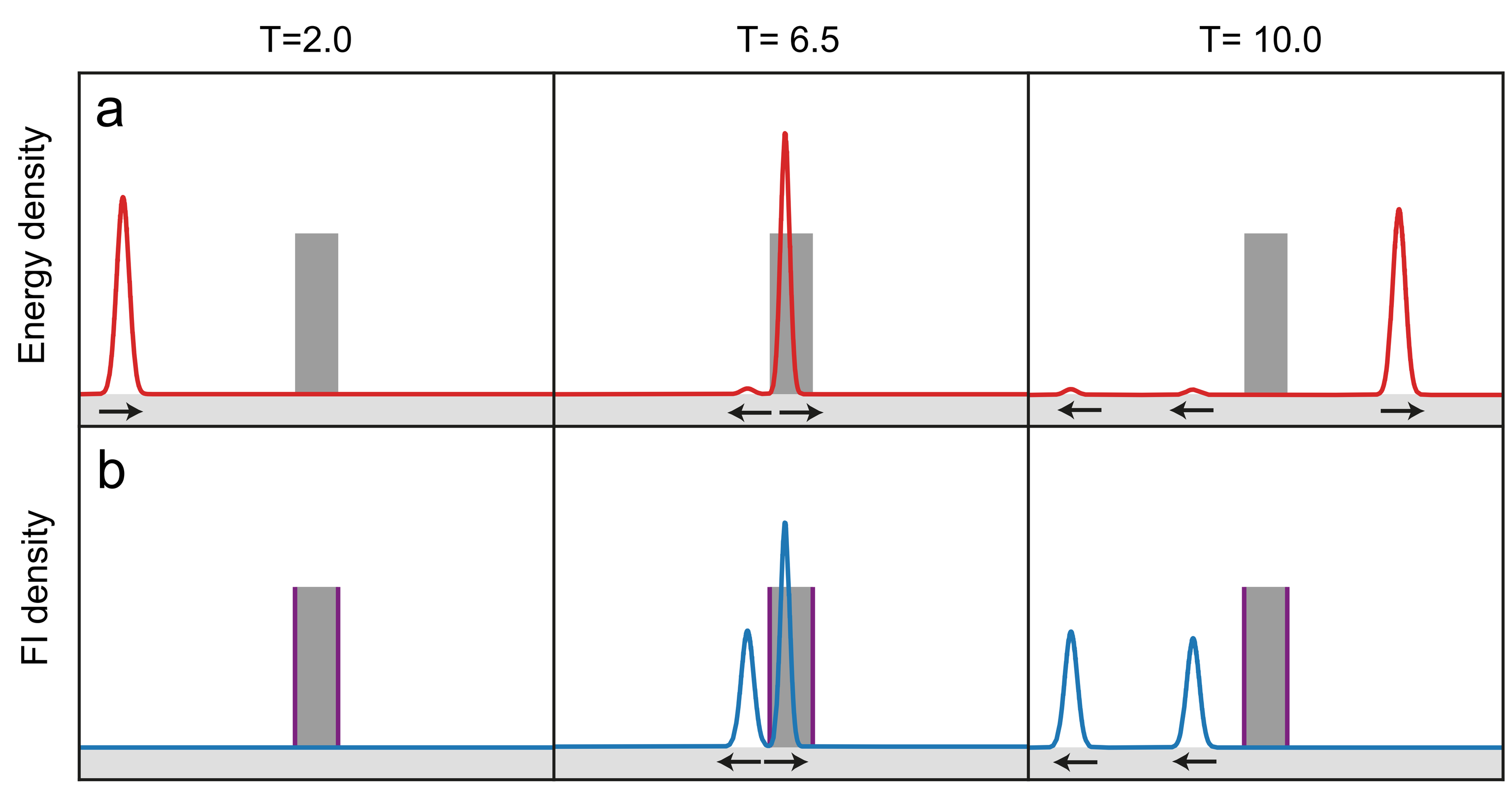
FI on the target's refractive index originates in regions sensitive to θ (purple). Here, the whole target scatter acts as a source of FI. The FI flows along the FI flux field (blue arrows) into the reflection channel.

Experiment and sources of FI



a Microwave setup of a thin waveguide with a complex scattering region (orange cylinders) illuminated by EM waves from the right. **b/c** The **Fisher information originates** in the parts of the system that are **sensitive to small changes of the parameter θ** (purple; here sides of the target).

FI in a wave packet



a An EM wavepacket enters the system from the left (red envelope) and hits the target around T=6.5; Most of the energy is transmitted. **b** Fisher Information on the target's position is **generated when the EM wave the θ sensitive regions** (purple; sides of the target). Because of destructive interference, no information on θ reaches the transmission channel.

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References:

[1] J. Hüpfl, F. Russo, L.M. Rachbauer, D. Bouchet, J. Lu, U. Kuhl, S. Rotter; Continuity equation for the flow of Fisher information in wave scattering. Nat. Phys. 20, 1294–1299 (2024).
[2] LIGO Scientific Collaboration and Virgo Collaboration; Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett. 116, 061102 (2016).
[3] C. Gonzalez-Ballestero, M. Aspelmeyer, L. Novotny, R. Quidant, O. Romero-Isart; Levitodynamics: Levitation and control of microscopic objects in vacuum. Science Vol 374, Issue 6564